 J. Fluid Mech. (1979), *vol.* 92, *part* 1, *pp.* 206–208
Printed in Great Britain

REVIEW

Fractals: Form, Chance and Dimension. By BENOIT B. MANDELBROT. W. H. Freeman and Co., 1977. 365 pp. $14.95.

How long is the coastline of Britain? How are the stars distributed? What is a velocity? These are but a few of the questions treated in this exceedingly interesting book, whose main theme can be stated as follows: For many physical objects the usual simple concepts such as smooth curves and surfaces, finite sets of points, etc., are quite inadequate. On occasions we need geometrical entities of an altogether different character which will correspond to the complexity, discontinuity or randomness of the phenomena to be described.

Historically the first such phenomenon was the Brownian motion, which inspired the construction of Wiener's Brownian functions $f(t)$ whose standard deviation increases like $t^{\frac{1}{2}}$. But many other mathematical entities regarded by physicists as either pathological or fit only for the pure mathematicians, have long been known. For example Cantor sets, Peano curves, Cauchy flights, and the like. It is the author's contention that these pure mathematical constructions, so long ignored by applied mathematicians, are precisely what is required today in many branches of physics and engineering. In his words: 'Mathematics is to be praised for having put these sets at our disposal long ago, and scolded for having discouraged us from using them'.

Aware of his audience, the author's method of presentation is to discuss first the salient features of each physical phenomenon, and only then to introduce the appropriate mathematical tool. Thus Chapter II, on measuring the length of a coastline, points out that the answer L will depend essentially on the scale η with which it is measured. On a scale of 10^2 m, for example, we see many bays and promontaries not visible on a scale of 10^3 or 10^4 m. Generally, L will increase as η diminishes. L. F. Richardson was the first to observe that L increases more rapidly than η^0 but less rapidly than η^{-1}. This leads to the idea that the dimension of L is equal to a number D lying between 1 and 2. In fact the appropriate mathematical tool for describing a coastline is some generalization of Koch's 'snowflake curve', a beautiful object discussed and tamed by Cesàro in 1905.

Thus gently the author introduces the idea of a 'fractal dimension' and derives the formula for self-similar fractals, namely

$$D = \ln N/\ln (1/r),$$

where r denotes the reduction in scale when the object is split into N similar parts. It soon appears that D is a special case of the Hausdorff–Besicovitch dimension of a generalized point-set. What is a fractal? According to the author's definition it is simply a point-set for which the Hausdorff dimension D is greater than its (ordinary) topological dimension D_T.

In this book, however, nearly all technical details and definitions are relegated to a final chapter, relying presumably on the plausible assumpton that many mathematicians who would run a mile from a Hausdorff measure will quite willingly fall into the arms of a fractal.

Chapter III next introduces the Brownian function and Brownian trail which are

0022-1120/79/0388-6510 $00.35

shown to have $D = 2$; also their zerosets, which have $D = \frac{1}{2}$. The following Chapter IV deals with errors in transmission lines. Such errors commonly occur in 'bursts', and are reasonably well modelled by Cantorian sets of points together with the intervals between them. *Their* dimension D lies between 0 and 1. To obtain good agreement with observation the order of the intervals must be shuffled at random.

Chapter V introduces a simple model for the clustering of stars, in which the mean density of matter is a decreasing function of the scale on which it is measured. Such a model provides a solution to Olbers' paradox: 'Why is it dark at night?' The 'curdling' theories of Hoyle and Fournier are also discussed. It is more difficult to believe in Rayleigh or Cauchy 'stop-overs' as a model for stellar clustering. Rayleigh flights, for instance, are essentially sequential. What sequence or order is involved in the set of instantaneous positions of stars?

For readers of the *Journal of Fluid Mechanics* one of the most interesting chapters will be that on turbulence (Chapter VI). The observed intermittency of many turbulent velocity fields certainly suggests that if a suitable inner scale is chosen, then zones of intense dissipation may be confined to point-sets of dimension $D < 3$. In fact Mandelbrot has shown that in that case the classical Kolmogorov exponent $\frac{5}{3}$ must be replaced by $\frac{5}{3}+B$, where $B = (3-D)/3$. Less convincing are the specific fractal models of the turbulence 'support' which are proposed in this chapter, for example the Sierpinski 'Sponge' and 'Pastry Shell'.

The above remark examplifies a possibly general criticism of the book, namely that the inclusion of a little more physics into the fractal models would probably enhance their correspondence to reality.

A similar comment seems to apply to the following chapter on geomorphology, where mountainous terrain is modelled by various fractals derived from Brownian functions. Though some of these do show a striking similarity to actual terrain, nevertheless the randomness here is evidently too great. The remedy may be not to *smooth* the model but to take account statistically of the *correlation* between different parts of the terrain, correlation probably introduced by the actual processes of erosion, for example the formation of continuously descending river valleys.

Subsequent chapters suggesting 'fractal' models for moon-craters, linguistic word-frequencies, and the Pareto law for salaries, seem to demonstrate the diversity of possible applications of fractals. But one cannot avoid the suspicion that the author would like to include in his province all stochastic processes, whether capable of a geometric representation or not.

The book ends with two important chapters. One of these contains several short biographical sketches of the pioneers of the subject, including particularly Georg Cantor (1884–1918), Louis Bachelier (1870–1946), Jean Perrin (1870–1942), L. F. Richardson (1881–1953); and the author's mentor Paul Lévy (1886–1971) though incidentally not G. I. Taylor or A. S. Besicovitch.

A final chapter entitled 'Mathematical Lexicon and Addenda' contains most of the technical definitions and theorems, and a few new suggestions, added since the original French version (*Les objets fractals: forme, hasard et dimension*, Paris 1975).

Altogether the book is to be praised for two outstanding features: the freshness and enthusiasm of the writing, particularly in those sections which are translations of the original French version. Secondly, the beautiful and copious illustrations, which for the most part are, and could only be, drawn with the aid of a computer. In stimulating and enlarging the imagination these drawings establish computer 'graphics' as a new art form.

It remains to pose the question: how valuable is the approach to physical problems which is represented by the present book? We have already mentioned the physical short-comings of some of the proposed models. To this the author will doubtless reply that the important thing is first to approach the phenomena from the point of view of fractals; then having found a suitable geometrical description the physics may later be added, inferred or simply ignored.

What is true certainly is that at this early stage some allowances must be made, that all successful models involve a judicious mixture of geometry and physical laws, and that this stimulating book introduces a variety of exciting new mathematical tools which should be brought to the attention of every applied mathematician. In the reviewer's opinion a course on 'Fractals for physicists' would be a valuable addition to the curriculum.

M. S. LONGUET-HIGGINS